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**INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION 2023
APPLIED MATHEMATICS-241**



CLASS : XII
DATE: 14-09-2023

TIME ALLOTTED : 3 HRS.
MAXIMUM MARKS: 80

GENERAL INSTRUCTIONS:

1. All questions are compulsory however in some questions of 2-, 3- and 6-marks internal choice is given. Students are expected to attempt any one out of the two.
2. Section A (Q-1 to Q -20) contains 20 one-mark questions.
3. Section B (Q-21 to Q -25) contains 5 two-mark questions.
4. Section C (Q-26 to Q -31) contains 6 three-mark questions
5. Section D (Q-32 to Q -34) contains 3 four-mark questions (Case study type)
6. Section E (Q-35 to Q -38) contains 4 five-mark questions.

1. If x is an odd integer, then $x^2 =$
a) $1 \bmod 8$ b) $2 \bmod 8$ c) $3 \bmod 8$ d) $5 \bmod 8$
2. $-39 \bmod 7 =$
a) 4 b) 2 c) 3 d) not possible to find
3. If $A = \begin{pmatrix} p & 0 \\ 1 & 1 \end{pmatrix}$, and $B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$, are two matrices, then $A^2 = B$ is true for
a) $p=1$ b) $p=-1$ c) $p=4$ d) no real value of p
4. P run 18 metre in the same time in which Q runs 20 metres . The distance by which Q beat P in a kilometre race is
a) 90 metre b) 80 metre c) 100 metre d) 900 metre
5. If $a, b \in \mathbb{R}$ and $a > b > 0$, then
a) $\frac{1}{a} > \frac{1}{b}$ b) $\frac{1}{a} < \frac{1}{b}$ c) $\frac{1}{a} \geq \frac{1}{b}$ d) $\frac{1}{a} \leq \frac{1}{b}$
6. $(5^{48} - x) \bmod 24$ is 0 then $x =$
a) 2 b) 0 c) 1 d) 23
7. The matrix $\begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{pmatrix}$ is a
a) diagonal matrix b) scalar matrix c) square matrix d) both b and c

8. If the mean of binomial distribution is 81 then the standard deviation lies in the interval
a) [0,9] b) (0, 3] c) [0, 81) d) [0, 9)
9. In a container, milk and water are present in the ratio 7 : 5, if 15 litre of water is added to this mixture the ratio becomes 7 : 8, then the quantity of milk in the original mixture is
a) 40L b) 38L c) 35 L d) 30L
10. While computing the trend values (using least square method) the values $a = \frac{\sum Y}{n}$ and $b = \frac{\sum XY}{\sum X^2}$ are computed from the normal equations by considering
a) $\sum Y = \sum X$ b) $\sum Y = 0$ c) $\sum X = 0$ d) $\sum X^2 = 0$
11. In hypothesis testing for a given degrees of freedom if the critical t value is less than the calculated t value then the null hypothesis is
a) rejected b) accepted c) cannot be determined d) none of these
12. For a Poisson Variate X, $P(X=k) = P(X=k+1)$ then variance of X is
a) k b) k - 1 c) k + 1 d) data not sufficient to calculate
13. If the first derivative of a function is given by $f'(x) = (x-1)^2(x-2)^3(3x-4)$, then f is strictly decreasing in
a) $(1, \frac{4}{3})$ b) $(\frac{4}{3}, 2)$ c) $(2, \infty)$ d) $(-\infty, 1)$
14. A machine makes car wheels and in a random sample of 26 wheels, the test statistics is found to be 3.07. As per the t-distribution (at 5% percent level of significance), the quality of wheels produced by the machine, if $[t_{25}(0.05) = 2.06]$, is
a) inferior b) same quality c) superior d) cannot be determined
15. If Z is a standard normal variate, then using the given table, the value of $P(|Z| > 1.3) =$
a) 0.0968 b) 0.1936 c) 0.8064 d) 0.4516
16. If matrix A and B are such that $AB = A$ and $BA = B$ then $(B^2 + B) =$
a) 2A b) O c) 2I d) 2B
17. Match the following columns to complete the sentence and choose the correct option

Trend component	Pattern of variation	Time period of variation
I. Secular trend	a. is a regular periodic variability	i. over a period more than a year
II. Cyclical trend	b. has smooth, regular variations	ii. within a period of one year
III. Seasonal trend	c. has oscillatory variation	iii. over a long term period

- (A) I - a - ii ; II - b - iii ; III - c - I (B) I - b - iii ; II - c - i ; III - a - ii
(C) I - b - ii ; II - c - i ; III - a - iii (D) I - b - ii ; II - a - iii ; III - c - i

18. If A is a square matrix and $\text{adj } A = \begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix}$, then matrix A =
a) $\begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -3 \\ -2 & 5 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ d) $\begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}$

19. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : For matrix $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$, $A(\text{adj } A) = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$,

Reason R : For a square matrix A , $A(\text{adj } A) = |A|I$

In the light of the above statements, choose the most appropriate answer from the options given below.

- (a) Both **A** and **R** are correct and **R** is the correct explanation of **A**.
 (b) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.
 (c) **A** is correct but **R** is not correct. (d) **A** is not correct but **R** is correct.

20. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : If a number exceeds its square by the greatest possible value, then the number is equal to $\frac{1}{2}$

Reason R : $f'(x) < 0$ for f to be maximum.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (a) Both **A** and **R** are correct and **R** is the correct explanation of **A**.
 (b) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.
 (c) **A** is correct but **R** is not correct. (d) **A** is not correct but **R** is correct.

21. A boat takes 10 hours to row 70 kilometres downstream and 14 hours to row the same distance upstream. Find the speed of the boat in still water and the speed of the stream.

OR

Find all integers x such that $1 \leq x \leq 30$ and $x \equiv 4 \pmod{5}$

22. Explain briefly and give one example each for the following sampling types:

- i) stratified sampling ii) systematic sampling

23. The following data is taken using simple random sampling; 5, 8, 10, 7, 10, 14.
 What is the point estimate of the population mean and population standard deviation?

24. If $y = x^x$ then prove that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$

OR

If $x = 4t - \frac{3}{t}$ and $y = 2 - \frac{1}{t}$, find $\frac{d^2y}{dx^2}$

25. If for a matrix $A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$, $A^2 - 3A - 7I = 0$, then find A^{-1}

26. The speed of a boat in still water is 15 km/hr. If the speed of the stream is 3 km/hr it takes one hour to go to a place and to come back. find the distance between two places.

27. The cost function for the manufacture of x number of goods by a company is

$$C(x) = x^3 - 9x^2 + 24x.$$

- i) Find the level of output at which the marginal cost is maximum.
 ii) Show that at output level 4.5 average cost will be minimum.

28. Using Cramer's rule check the consistency of the following system of equations.

$$\begin{array}{ll} \text{i) } 5x - 10y = 4 & \text{ii) } 4x - y = 3 \\ x - 2y = 8 & 12x - 3y = 9 \end{array}$$

OR

Given the following equations for two related markets A and B, find the equilibrium conditions for each market and the price for each by Cramer's rule:

$$x_d(A) = 82 - 3p_A + p_B ; \quad x_s(A) = -5 + 15p_A$$

$$x_d(B) = 92 + 2p_A - 4p_B ; \quad x_s(B) = -6 + 32p_B$$

where x_d and x_s denotes the quantity demanded and quantity supplied respectively and p_A and p_B represents the price for each market.

29. Find the intervals in which the function $f(x) = 2x^3 - 24x + 107$ is

- (i) strictly increasing (ii) strictly decreasing

30. A class 12 twelve student wish to apply some colleges. If X denotes the number of colleges where he can apply after the result and $P(X=x)$ denotes her probability of getting admission in x number of colleges. Its given that

$$P(X=x) = \begin{cases} kx, & \text{if } x = 0 \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \end{cases}, \text{ where } k \text{ is a positive constant.}$$

- (i) Find the value of k.
(ii) What is the probability that a student will get admission in at most 2 colleges?
(iii) Find the variance of the probability distribution

OR

An airline accepts the reservation for the seats on a particular flight of a 98-seater aircraft. From the past experience its known that 3% of the people who reserve seats do not turn up so the airline has a policy of allowing 100 persons to book the seats on the flight. What is the probability of more than 98 persons turning up for the flight. **Given: $e^{-3} = 0.05$**

31. Mr. Vikas runs a bread factory and the record of his sales of bakery items for the period of 2015 - 2019 is as follows:

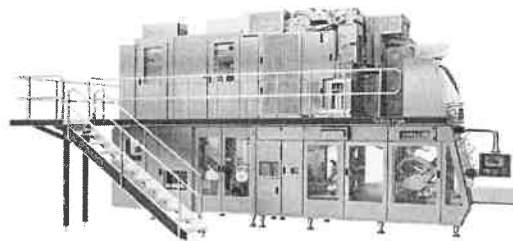
Year	2015	2016	2017	2018	2019
Sales (in ₹ thousands)	35	42	46	41	48

Use the method of 3 years moving averages to smoothen the data. Depict it graphically.

32. A swimming pool is filled by 3 pipes A, B & C with a uniform flow. The first 2 pipes A and B operating simultaneously fill the pool in the same time during which the pool is filled by the pipe C alone. The second pipe B fills the pool 5 hours faster than the first pipe and 4 hrs slower than the 3rd pipe C. If a, b and c are the times taken(in hrs) by the three pipes A, B and C then
- i) show that $\frac{1}{a} + \frac{1}{a-5} = \frac{1}{a-9}$
- ii) find the time required by the first pipe to fill the swimming pool.



33. A machine is designed to fill 500 ml of milk to polythene bags. A randomly selected 25 milk bags filled by this machine are inspected. The mean of milk quantity is found to be 498 ml and SD is 10 ml.
- Establish the null and alternate hypothesis
 - Calculate the test statistic and compare with the critical value
 - Justify if the machine is functioning properly a 5% level of significance?



Given: $t_{24}(0.05) = 2.391$

$t_{24}(0.01) = 3.091$

- OR**
- Justify if the machine is functioning properly a 1% level of significance?
34. If the heights of 1000 students are normally distributed with mean 173 cm and standard deviation of 7.5 cm then
- how many students have heights
 - greater than 183 cm
 - less than 162.5 cm
 - 75% of the students have heights less than h cm. Find h .



- OR**
- If 80% of the students have height more than H cm, find H
(refer z-table on the last page)
35. If matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 2 & 2 \end{pmatrix}$, find A^{-1} . Hence solve the following system of equations

$$x + 2y + z = 8, \quad 2x + 3y + 2z = 14, \quad 3x + 2y + 2z = 13$$

36. For 6 trials of an experiment, let X be a binomial variate which satisfies a relation: $9P(x = 4) = P(X = 2)$. Find the probability of success, mean and variance of the distribution.

OR

If mean and variance of a binomial distribution are 2 and $\frac{4}{3}$ respectively, find

- the probability of exactly 2 success.
 - the probability of atleast 1 success.
37. The production manager of a company plans to include 180 sq. cms. of actual printed matter in each page of a book under production. Each page should have a 2.5 cm wide margin along the top and bottom and 2 cm wide margin along the sides. What are the most economical dimensions of each printed page?

OR

The price per unit for a commodity is $p = 100 + x$ and cost function is

$$C(x) = \frac{1}{3}x^3 - 7x^2 + 111x + 50.$$

- Find the profit function
- Find the number of units at which profit will be maximum.

38. The following table relates to the tourist arrivals (in millions) during 2004 to 2010 in India:

Year (T)	2004	2005	2006	2007	2008	2009	2010
Tourist arrivals(Y)	18	20	23	25	24	28	30

Using method of least squares

- (i) Find the line of best fit
(ii) Predict the number of tourist for the year 2014.

OR

Calculate the 4 year moving average for the number of tourist arrivals (in millions) during 2004 to 2011 in India.

Year (T)	2004	2005	2006	2007	2008	2009	2010	2011	2012
Tourist arrivals(Y)	18	20	23	25	24	28	30	32	35

An extract of Z - Distribution Table

$$P(z < a)$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

****END OF THE QUESTION PAPER****